CHAPTER 12

DIFFERENTIAL THRESHOLDS, WEBER FRACTIONS AND JND'S

THE ANALOGS

Before pressing into this rather detailed chapter, let's take a minute for purposes of orientation. In the flow diagram, Figure 1.2, we are still within the center block: evaluation of $F = kH$. As I have reiterated at various times, this book is largely about the unification of the laws of sensation. I am endeavoring to show that the single equation, $F = kH$, with a single, explicit mathematical form for the H-function, will encompass all sensory phenomena involving the variables steady I, F, and $t > 30$ ms. for a single stimulus. While we could possibly improve our curve-fitting, upon occasion, by modifying the H-function in some *ad hoc* manner, that is not really the issue. The important matter is that *one* function permits us to account for (nearly) *all* observed sensory effects in a quantitative manner, and from this *one* function we can derive (nearly) *all* the empirical laws that have been formulated during the past 130 years. Unification is not a game to be played for the exercise, or for its own sake (to say "I climbed the mountain"). Unification is pursued for the physical and biological insight provided by the unifying equation, as we discussed in Chapter 1, as well as for its predictive value.

We recall the analog to the ideal gas law that was suggested in the first and third chapters:

$$\begin{matrix} V & T & P \\ \Updownarrow & \Updownarrow & \Updownarrow \\ t & I & F \end{matrix}$$

(i)

$$P \propto T \qquad \text{or} \qquad P = f(T, V'),$$

(Charles' law) : pressure is a monotone increasing function of T with $V = V' =$ constant, analogous to

$$F = F(I, t'),$$

(Law of sensation) : F is a monotone increasing function of I with $t = t'$ =constant.

(ii)

$$P \propto 1/V \qquad \text{or} \qquad P = f(T', V),$$

(Boyle's law) : Pressure is a power function of V with $T = T' =$ constant, analogous to

$$F = F(I', t),$$

(Law of adaptation) : F is a power function of t (for larger t) with $I = I' =$ constant.

In this chapter we come to the third equation:

(iii)

$$\Delta T/T \propto \Delta P \cdot (1/T)$$

with ΔP held constant.

We shall now show that

$$\Delta I / I \propto \Delta F \cdot (1/I) \text{ with } \Delta F \text{ held constant (well, almost!).}$$

No. There is no profound connection (of which I am aware) between the ideal gas law and the entropy law. I am just trying to illustrate, particularly for readers who are unfamiliar with the methods of physics, how a single equation can contain within itself the explanation for many, apparently diverse physical phenomena. I am trying to encourage the reader, in this way, to think of the law of sensation, the principle of adaptation, and the Weber fraction as different views of the same principle of perception: to perceive is to gain information.

THE WEBER FRACTION

The Weber fraction, $\Delta I / I$, was introduced in Chapter 3, The Weber Fraction, and that material should probably be reviewed at this time. We require a detailed understanding of virtually every paragraph in that section as we proceed through the following theoretical derivation of the mathematical function for $\Delta I / I$.

The mathematical technique that will be employed is to replace the differentials in a differential equation by their finite differences (e.g. replace dx by Δx), the inverse of the process used by Fechner (Equations (3.2) and (3.3)), who replaced finite differences by the corresponding differentials. We are already in possession of an integrated function, the *H*- or *F*-function, and we shall proceed to the corresponding differential equation; Fechner began with a differential equation and integrated to obtain his "*F*-function."

Luce and Edwards (1958) supported by Krantz (1971) and others attempted to show that Fechnerian integration was in error, and that jnd's could not, in general, be summated using Equations (3.11) and (3.12). In the present discussion, we confine Δx to be small in comparison with x, and the objections expressed mathematically on page 225 of the paper by Luce and Edwards do not apply. It is important to understand and to observe that total jnd's calculated from Equations (3.11) and (3.12) is, indeed, nearly the same value as obtained by adding jnd's one on top of the other, the method recommended by these authors (Luce and Edwards, page 233). The reader is referred to Figures 3.5(a) and (b), showing the data of Lemberger (1908), dealing with differential thresholds for sugars. These data were taken from Table 4 of Lemberger's paper, which is reproduced here, in part, in Table 12.1

Table 12.1 Data of Lemberger (1908): Weber fraction for sucrose

Measured Differential Threshold (Conc. of soln in percent)	Weber Fraction	Measured Diff Thresh (Conc. in Percent)	Weber Fraction
0.44 – 0.60	0.3636	5.080 – 5.833	0.1482
0.60 – 0.82	0.3666	5.833 – 6.750	0.1572
0.820 – 1.125	0.3720	6.75 – 7.75	0.1481
1.125 – 1.400	0.2444	7.750 – 8.916	0.1505
1.40 – 1.63	0.1643	8.916 – 10.35	0.1608
1.63 – 1.85	0.1350	10.35 – 11.97	0.1565
1.85 – 2.13	0.1514	11.97 – 13.90	0.1612
2.13 – 2.45	0.1502	13.90 – 16.17	0.1633
2.450 – 2.825	0.1531	16.17 – 18.75	0.1596
2.825 – 3.267	0.1561		
3.267 – 3.775	0.1558		
3.775 – 4.390	0.1629		
4.39 – 5.08	0.1572		

Note. German, "Saccharose" translated as "sucrose"

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(last few points omitted). In Chapter 3, with reference to Figure 3.5 (b), and using Equation (3.12), we calculated the total number of jnd's from I = threshold to I = 18.75% solution to be 21 jnd's. However, as may be seen from Table 12.1, Lemberger actually made 22 measurements of $\Delta I / I$ by adding jnd's one on top of the other. That is, $(\Delta I)_1$ extended from 0.44% to 0.60% solution; $(\Delta I)_2$ extended from 0.60% to 0.82% solution, etc. So the 22 measurements, extending from 0.44% to 18.75%, comprised exactly 22 jnd's. This value agrees exceedingly well with our value of 21 jnd's obtained using Equation (3.12). However, the values of ΔI were small compared with I. When ΔI approaches I in value, one must, indeed, heed the warning of Luce *et al.*

The above having been said, we proceed with the derivation of the theoretical function for the Weber fraction, $\Delta I / I$, which issues directly from Equation (9.19),

$$H = \tfrac{1}{2} \ln(1 + \beta I^n / t) \; . \tag{12.1}$$

It will be assumed that in experiments for determining Weber fractions, all stimuli are applied for the same interval of time, t', so that we may again introduce Equation (10.2)

$$\gamma = \beta / t' \; . \tag{12.2}$$

Thus

$$H = \tfrac{1}{2} \ln(1 + \gamma I^n) \; . \tag{10.3) / (12.3}$$

Differentiating H with respect to I,

$$\frac{dH}{dI} = \frac{\tfrac{1}{2} \gamma n I^{n-1}}{1 + \gamma I^n} \; . \tag{12.4}$$

Replacing dH and dI by the corresponding finite differences, and rearranging the equation,

$$\Delta I = \frac{2\Delta H (1 + \gamma I^n)}{\gamma n I^{n-1}} = \frac{2\Delta H}{n} \left[I + \frac{1}{\gamma I^{n-1}} \right] . \tag{12.5}$$

Dividing both sides by I,

$$\boxed{\Delta I / I = \frac{2\Delta H}{n} \left[1 + \frac{1}{\gamma I^n} \right] .} \tag{12.6}$$

The above equation still makes no physiologically meaningful statement; it simply relates a change, ΔI, in stimulus intensity, to a corresponding change, ΔH, in entropy. There is nothing yet to render $\Delta I / I$ interpretable as a Weber fraction. We are interested in the *stimulus change per jnd*. Therefore, it is necessary to add an additional assumption to the list of 6 assumptions given in Chapter 9. I recommend

(7) The subjective magnitudes of all jnd's are equal.

The reader will recall that this assumption was originally due to Fechner [(Equation (3.2)]. We do not, however, require the full complement of 7 assumptions for a description of the Weber fraction because, due to the introduction of Equation (12.2), we have, effectively, dropped assumption (4). Assumption (7) can, if desired, be relaxed (please see "ΔH as threshold," below).

In the discussion that follows, we associate each stimulus of magnitude, I, with a unique quantity of information, the information that it could transmit to the receptor if the receptor were completely unadapted to the stimulus; that is, the potential information of the stimulus. This concept was introduced in the previous chapter, augmented by note 5 in that chapter. There is, further, a double-stimulus approximation discussed below ("On the Physical Meaning of ΔI ").

Letting the constant subjective magnitude of the jnd be ΔF, we can divide both sides of Equation (12.6) by this quantity to give

$$\frac{\Delta I / \Delta F}{I} = \frac{2\Delta H / \Delta F}{n} \left[1 + \frac{1}{\gamma I^n} \right] , \tag{12.7}$$

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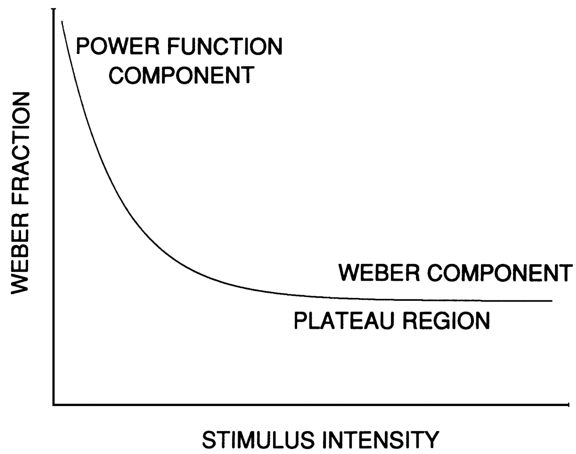

Figure 12.1 Weber fraction, $\Delta I/I$, vs. stimulus intensity, I. Illustrates the basic shape of the curve measured for various sensory modalities. $\Delta I/I$ is large for small values of I (even though ΔI is quite small). This component of the curve has been labeled "power function component" because it is described in Equation (12.6) by means of an I^n-term. Some investigators have recorded only this part of the curve, and report a monotonically decreasing value for $\Delta I/I$. For larger values of I, $\Delta I/I$ approaches a constant. This component has been called the "Weber component" because Weber believed that $\Delta I/I$ was universally constant. Curves of exactly this shape are seldom measured, but were, in fact, recorded by Knudsen and Riesz for audition. The general shape is also found for taste receptors of insects. It is described by Equations (12.6) and (12.16). A more usual shape for the Weber fraction curve is shown in Figure 12.2.

or, since

$$\Delta F = k\Delta H\,, \tag{12.8}$$

$$\frac{\Delta I/\Delta F}{I} = \frac{2}{kn}\left[1 + \frac{1}{\gamma I^n}\right]. \tag{12.9}$$

The expression $(\Delta I/\Delta F)/I$, introduced in Chapter 3, then, represents the Weber fraction in a complete fashion: fractional change in stimulus intensity per jnd. However, we shall usually use the form (12.6). ΔH is the *informational differential threshold*. Note that we have now incorporated both of Resnikoff's requirements (Chapter 10).

With ΔH $(= \Delta F/k)$ taken as the constant informational cost of a jnd by assumption 7, above, Equation (12.6) is a simple expression giving $\Delta I/I$ as a function of I. Equation (12.9), by the way, is about as close as we get to the third analog of the ideal gas ((iii) above).

Equations (12.6), (12.7) or (12.9) define a theoretically derived function that describes the observed shape of many Weber fraction curves. For smaller I, the second term on the right-hand side dominates and $\Delta I/I$ is large. We must be careful, though, because as the fraction becomes large, $\Delta I \to I$, and the finite difference approximation weakens. As I becomes greater, the second term on the right-hand side approaches zero, and $\Delta I/I$ approaches $2\Delta H/n$. That is,

$$\text{Weber constant} = 2\Delta H/n\,. \tag{12.10}$$

The general form of the function (12.6) is illustrated in Figure 12.1. The early, rising portion of the curve is derived from the term γI^n, so it might be called the *power function component*; the later plateau in the curve, corresponds to Weber's law, Equation (3.1),

$$\Delta I\,/\,I = \text{constant},$$

so it might be called the *Weber component*. The shape of these curves, early rise followed by plateau, is of the type measured by Riesz (1928) for audition. Equation (12.6) cannot, however, describe the *terminal rise* in the curve, the rise observed for larger values of I, as seen, for example, in the curves measured by König for vision (Figure 3.6), or Holway and Hurvich for taste of sodium chloride (Figure 12.2).

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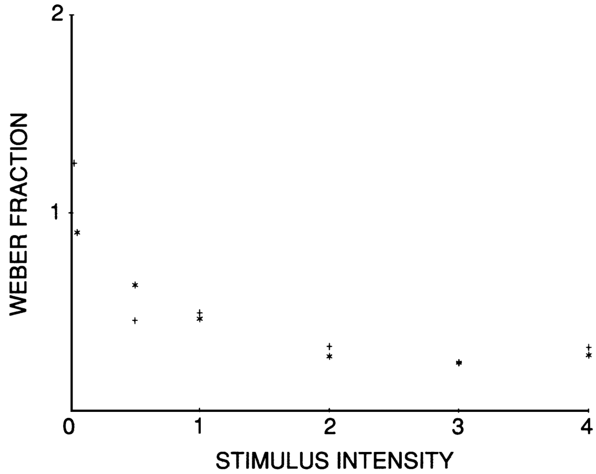

Figure 12.2 $\Delta I/I$ vs. I for taste of sodium chloride. The oft-cited data of Holway and Hurvich (1937). Although no curve has been drawn in, the data conform to a curve of the type shown in Figures 3.5a amd 3.6.

I can trace the history of the use of equations like (12.6) back as far as 1907, when a very similar equation was used empirically by Nutting:

$$\Delta I / I = P_m + (1 - P_m)(I_0 / I)^n . \tag{12.1}$$

It was also used, in the same form, by Knudsen (1923) and an equation of exactly the same type as (12.6) was used empirically by Riesz (1928). In the present work, Equation (12.6) was derived from the general equation of entropy (12.1).

Békésy (1930) modeled the neural excitation process to obtain the equation

$$E = b\log(1 + \tfrac{a}{c}J) , \tag{12.12}$$

where E is "excitation" (potential), c is tissue salt concentration, J is sound pressure, and b is constant, as is a/c. This equation is very close to our equation of entropy. Békésy gives credit to Alfred Lehmann (1905)[1] for the original derivation of Equation (12.12). Békésy then derived from Equation (12.12) our Equation (12.6), with $n = 1$ and, referring to Knudsen's auditory data, showed that the measured Weber fraction for frequencies of 200 and 1000 Hz were well fitted by this equation. Békésy's model is summarized by Harris (1963). In Chapter 3, we followed Ekman's derivation of the $n = 1$ equation (Equation (3.10)) and we learned that this equation may date back as far as Fechner himself.

Equation (12.4) can be cast into a new and useful form. From Equation (12.3),

$$1 + \gamma I^n = e^{2H} . \tag{12.13}$$

Since

$$\tfrac{1}{2}\gamma n I^{n-1} = \tfrac{1}{2}(n/I)(\gamma I^n) = \tfrac{1}{2}(n/I)(1 + \gamma I^n) - \tfrac{1}{2}(n/I) ,$$

therefore

$$\tfrac{1}{2}\gamma n I^{n-1} = \tfrac{1}{2}(n/I)(e^{2H} - 1) . \tag{12.14}$$

Introducing Equations (12.13) and (12.14) into (12.4),

$$\frac{dH}{dI} = \tfrac{1}{2}(n/I)\,\frac{e^{2H} - 1}{e^{2H}}$$

or

$$\boxed{\frac{dH}{dI} = \tfrac{1}{2}(n/I)(1 - e^{-2H}) .} \tag{12.15}$$

Again, going over into the Δ-form,

$$\boxed{\Delta I / I = \frac{2\Delta H}{n}\left[\frac{1}{1 - e^{-2H}}\right].} \tag{12.16}$$

This interesting equation gives the Weber fraction in terms only of the variable, H, or since $H = F/k$, in terms of F. For example, for audition, $\Delta I/I$ is given solely in terms of loudness. Notice that as loudness, H, becomes large, $\Delta I/I \to 2\Delta H/n$ = *Weber's constant*. As loudness decreases, $\Delta I/I$ becomes larger, as seen in Figure 12.1 (Riesz type). Equations (12.6) and (12.16) are mathematically equivalent.

Equation (12.15) is, perhaps, the most natural *differential equation for the entropy*. This equation can be solved, by separating the variables, to give the entropy function, H (Equation (12.1)). The constant γ emerges as a constant of integration.

DERIVATION OF THE POULTON-TEGHTSOONIAN LAW

This law (PT law henceforth), in which exponent plotted vs. stimulus range defines a rectangular hyperbola, has been described in Chapter 3, Equation (3.27), and this is the perfect time to review it. I think there is no better example of the power of the entropy equation than its ability of produce the PT law in its totality, including the value of the constant. The law can actually be obtained, in part, by very simple means, which I shall give first, followed by a second, somewhat more lengthy derivation which provides the complete form of the law.

First, rather simply, writing the H-function in the "Fechner" or semilog approximation (Equation (10.5)),

$$H = \tfrac{1}{2} n \ln I + \tfrac{1}{2} \ln \gamma . \tag{12.17}$$

We recall that this approximation is valid only for $\gamma I^n >> 1$. Let us take two values for I, one close to the physiological maximum, I_{high}, and one close to the minimum value for which (12.17) is valid, I_{low}. Then

$$H_{\text{high}} = \tfrac{1}{2} n \ln I_{\text{high}} + \tfrac{1}{2} \ln \gamma , \tag{12.18a}$$

and

$$H_{\text{low}} = \tfrac{1}{2} n \ln I_{\text{low}} + \tfrac{1}{2} \ln \gamma . \tag{12.18b}$$

Then

$$\delta H = H_{\text{high}} - H_{\text{low}} , \tag{12.19}$$

or

$$\delta H = \frac{1}{2} n \ln I_{\text{high}} / I_{\text{low}} , \tag{12.20}$$

from which, if I_{low} is "low enough," we have approximately,

$$n \cdot \ln \text{(range of intensities)} = 2\, \delta H . \tag{12.21}$$

We see that Equation (12.21) does describe the PT law [Equation (3.27)] formally, without, however, providing a value for the quantity $2\delta H$ on the right-hand side. The reader can evaluate δH using Equation (12.19) with the assumption that $H_{\text{high}} >> H_{\text{low}}$.

The second derivation will achieve the same end using the concept of the jnd. Let us return to Equation (3.14):

$$\delta N = N_{\text{plateau}} = \frac{\ln 10}{\text{Weber constant}} \log_{10}(I_{\text{high}} / I_{\text{low}}) , \tag{12.22}$$

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where δN is the number of jnd's contained beneath the plateau or Weber-region of the Weber fraction curve. In the above equation we insert Equation (12.10), giving

$$\delta N = \frac{n \ln 10}{2\Delta H} \log_{10}(I_{\text{high}} / I_{\text{low}}) . \tag{12.23}$$

Let H_{high} now be defined as the entropy of the maximum physiological stimulus, calculated from Equation (12.3) (no approximation is used). Define H_{low} as the entropy, calculated from Equation (12.3) of the stimulus whose intensity marks the lower end of the plateau region of the Weber fraction. As before (12.19),

$$\delta H = H_{\text{high}} - H_{\text{low}} , \tag{12.24}$$

but with the quantities on the right-hand side defined differently (now with respect to a plateau). Then, since ΔH is the constant entropy span of one jnd (by assumption 7),

$$\delta N = \delta H \,/\, \Delta H . \tag{12.25}$$

That is, we could "fit in" $\delta H / \Delta H$ distinguishable stimuli beneath the plateau (see also Note 3). Equations (12.23) and (12.25) now each provide expressions for δN. Equating the right-hand sides of these two equations, and canceling ΔH from both sides,

$$\delta H \text{ (natural units)} = \frac{n \ln 10}{2} \log_{10}(I_{\text{high}} / I_{\text{low}}) . \tag{12.26}$$

Now, $\log_{10}(I_{\text{high}} / I_{\text{low}})$ is the quantity defined for Equation (3.27) as the $\log_{10}$ *range* of stimuli spanning the entire Weber fraction curve. Here we define this quantity as the range of stimuli spanning the plateau region of the Weber fraction curve. Rearranging Equation (12.26),

$$(n)(\log_{10} \text{range}) = 2\, \delta H \,/\, \ln 10 , \tag{12.27}$$

so that we have, again, nearly derived Equation (3.27) except for the value of δH. The advantage, however, in this second derivation, through the medium, as it were, of the Weber fraction, is that we can provide *a priori*, an approximate value for δH.

The clue lies in Equation (12.25). Since ΔH, the informational value of the jnd, is constant, δH, is proportional to δN, the total number of jnd's under the plateau of the Weber fraction. But we learned, by experience, that the number of jnd's beneath the plateau is not a bad approximation of the *total* number of jnd's, threshold to maximum physiological stimulus. For example (Chapter 3, The Weber Fraction), in the analysis of Lemberger's data on the differential threshold of taste, there are 19.8 jnd's beneath the plateau, and 21 jnd's under the whole of the analyzed curve (3 final values omitted). In general, fewer that 30% of the total jnd's lie to the left of the plateau,[2] so that

$$\delta N_{\text{plateau}} \text{ is less than but approximately equal to } \delta N_{\text{total}} . \tag{12.28}$$

Therefore, from Equation (12.24), δH, which is defined equal to $\delta H_{\text{plateau}}$, is less than but approximately equal to δH_{total}. But since H_{low} is small,

$$\delta H_{\text{total}} \simeq H_{\text{high}} , \tag{12.29}$$

and H_{high} has been shown to be a measure of the channel capacity of the modality, particularly in those senses that adapt completely (Chapter 11). Therefore, with an element of approximation

$$\delta H_{\text{total}} \simeq 2.5 \text{ bits } = 1.75 \text{ natural units of information,} \tag{12.30}$$

where the 2.5 bits is the universal channel capacity ($\simeq \log_2$ of the "magical number" 6). (The same value can be inserted for δH in Equation (12.21) to complete the first derivation of the PT law.)

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Inserting this value for δH into Equation (12.27),

$$(n)(\log_{10}\text{ range}) = \frac{(2)(1.75)}{\ln(10)} = 1.52\,, \tag{12.31}$$

which is the PT law in a form very close to that discovered empirically by Teghtsoonian using data assembled by Poulton. Since the value used for δH was a little too small (we used just the range of stimuli spanned by the plateau region rather than the total stimulus range), we could really expect that the "correct" value for the constant would be 15-30% larger than 1.52. The near-perfect agreement is probably fortuitous.

Note that Equation (12.31) is expected to be valid across the modalities, because of the nearly constant value of the information content of a stimulus: 2.5 bits = 1.75 n.u. That is, (12.31) is an inter-modality law. Whether it is an *intra*modality law, holding within a modality where n changes, for example with frequency, remains to be thought through.

INFORMATION PER STIMULUS FROM WEBER FRACTIONS

In order to derive the PT law, we inserted an average value for the information per stimulus (channel capacity) of 2.5 bits per stimulus. It is instructive, now, to proceed without using the average value, but rather to calculate the stimulus information for each modality. There's nothing really new here; we are just turning the problem around. Putting Equation (12.26) into words, we have

$$\text{Information per stimulus (bits)} = \frac{\ln 10}{2\ln 2}\,(\text{power function exponent}) \cdot (\log_{10}\text{ stimulus range of plateau})\,. \tag{12.32}$$

If we now eliminate ($\log_{10}$ stimulus range of plateau) using Equation (12.22),
Information per stimulus (bits)

$$= \frac{1}{2\ln 2}\,(\text{power function exponent})(\text{jnds beneath plateau}) \cdot (\text{Weber constant})\,. \tag{12.33}$$

Since both of the above two equations utilize the plateau region of the Weber fraction, and the plateau is, in practise, often difficult to define, it would be useful to find an equation for stimulus information that does not require definition of the plateau. Such an equation was derived by the author (1987):
Information per stimulus (bits)

$$= \frac{\ln 10}{2\ln 2}\,(\text{power function exponent})(\log_{10}\text{ total stimulus range}) - \frac{1}{2\ln 2}\ln\left[\frac{\text{Weber fraction just above threshold}}{\text{Weber fraction at maximum stimulus}}\right]. \tag{12.34}$$

However, in eliminating the need to define the plateau in Equation (12.34) we may have gained very little. We must now evaluate the Weber fraction close to threshold, where measurements are not very accurate. Moreover, in the derivation of (12.34), the usual $dI \to \Delta I$ has been used, which is barely acceptable for larger ΔI.

Willy nilly, Equations (12.32), (12.33) and (12.34) are evaluated for 3 modalities: taste, vision and audition in Table 1 of Norwich (1987). I think that all 3 equations fare reasonably well in providing values for information transmitted per stimulus, but Equation (12.34) seems to be the weakest.

INVARIANTS IN MEASURING DIFFERENTIAL THRESHOLDS

We recall from Chapter 3 that measurement of the magnitude of the jnd was dependent upon the statistical criterion selected by the experimenter (Figure 3.7). That is, there is no unique measurement

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of the size of the jnd. One might, then, ask how variables such as "total number of jnd's beneath plateau" can be used as a variable in an equation. The answer lies in Equation (3.14):

$$(\delta N)\,(\text{Weber constant}) = \ln 10 \cdot \log_{10}(\text{range of stimuli})\ . \qquad (3.14)$$

The right-hand side of this equation is independent of the measurement of the jnd. The left-hand side contains the product (δN)(Weber constant). This product is, then, an invariant, which is independent of the statistical criterion used to measure the jnd. Remember that the Weber constant is equal to the value of $\Delta I / I$ under the plateau. As the criterion for measurement of the jnd becomes more lax (for example, the 75% criterion of Chapter 3 becomes, say, 50%), the magnitude of the jnd, ΔI, decreases, so that the total number of jnd's, δN, increases. But the product, $(\delta N)(\Delta I)$, remains invariant. That is, δN can enter as a variable into a general equation governing sensory function as long as it is multiplied by a "balancing" factor, such as ΔI or the Weber constant.

ON THE PHYSICAL MEANING OF ΔI†

One should understand clearly the distinction between ΔI and σ, as we have used them. ΔI represents a change in the *mean* value of the intensity of a stimulus signal. σ^2 represents a steady state fluctuation in the *instantaneous* value of a stimulus, and is brought about by quantum effects and internal biological variations [as discussed in Chapter 9, "Relationship between Variance and Mean" (refer also to note 3 of Chapter 9)]. We, as investigators, can control ΔI; we have no direct control over σ^2.

There is another very important distinction that must be made — this one concerning the manner or mode in which ΔI is produced by the experimenter. This distinction is made very clearly for the case of audition by Viemeister (1988, Fig 1), and for vision by Cornsweet and Pinsker (1965, Fig 2). In one mode, the subject must detect which of two stimuli, I or $I + \Delta I$, is the more intense. In a second mode, the subject must detect a brief change, ΔI, in a continuously administered stimulus. It is the first of the two modes that is encoded by our mathematical derivation of the Weber fraction. That is, we incremented I by ΔI and took account or the corresponding increment in H, ΔH, to obtain Equation (12.6). The method of Riesz is related to, but not exactly the same as, the second mode. My colleagues and I have treated the second mode mathematically, in a preliminary manner, in a series of publications cited below. However, in this book, we shall be concerned exclusively with the first of the two modes.

In both modes, in effect, two stimuli are administered, the second stimulus to a subject who may be in a partially adapted state. This state of partial adaptation is *not* allowed for in our derivation of Equation (12.6), which may introduce an error.

THE TERMINAL, RISING PORTION OF THE WEBER FRACTION CURVE

A number of investigators have reported measurements showing that the graph of $\Delta I/I$ vs. I either approaches a plateau (Figure 12.1) or declines monotonically with increasing I, and we have treated this case theoretically in Equations (12.6) or (12.16), corresponding to the first mode of stimulus administration. However, measurements of $\Delta I/I$ more usually show a "terminal rise." That is, rather than descending to a plateau, the curves fall in a gentle arc, and then rise again at higher values of I (see Figure 3.6). This characteristic shape of the curve is common to many modalities of sensation. An explanation for this terminal rise, based on the first mode of stimulus administration, was the substance of a Masters thesis submitted by my student, Kristiina Valter McConville (1988). The theory was also reviewed by Norwich (1991). However, because of its complexity, and because the values of the parameters t_o and β required to make the theoretical equations match the observed experimental data differed from the values of these parameters required in other equations, I have decided to omit the derivation here. So, for the moment, we shall continue to regard the terminal rise of the Weber fraction as a bit of a mystery, or, perhaps due to a saturation effect as one approaches the maximum physiological stimulus.

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ΔH AS THRESHOLD

The use, in this chapter, of the quantity ΔH to define a fixed quantity of information transmitted for each jnd may be viewed as a modernized version of Fechner's conjecture of constant ΔF (difference in subjective magnitude) for a jnd.[3] We have, in effect, used ΔH as a *threshold for discrimination.*[4] However, it is not, by any means, certain that $\Delta H = \Delta F / k =$ constant is a valid statement.

A very clever technique for measuring the subjective magnitude of the jnd was devised by Stevens (1936) in a paper that is seldom cited. Stevens measured loudness (subjective magnitude) as a function of sound intensity; and he used Riesz's data (1933) to obtain total jnd's as a function of sound intensity. He was then able to plot total jnd's against loudness, and found the relation (using F for loudness and N for number of jnd's)

$$F = K N^{2.2}, \qquad K \text{ constant} \tag{12.35}$$

or

$$\frac{dF}{dN} \rightarrow \frac{\Delta F}{\Delta N} = 2.2\, K N^{1.2}. \tag{12.36}$$

That is, taking $\Delta N = 1$ jnd, the increment in subjective magnitude, ΔF, increased as $N^{1.2}$. That is, ΔF was not constant for hearing.

However, the jury is still out on this matter. I would like to see Stevens-type, dual experiments performed with the same subjects: the first experiment F vs. I, and the second ΔI vs. I to give N vs. I. Such dual experiments should be performed for a number of sensory modalities. From each pair of experiments one could obtain plots of N vs. F, which would speak to the issue of the constancy of the jnd. Assumption 7 ("The Weber Fraction," above) can be replaced by any equivalent statement of the form: $\Delta F =$ experimentally determined function of I, giving rise to a variation of Equation (12.6).

WORKING TOWARD A COMMON SET OF PARAMETERS

It would be desirable, for each modality, to obtain a unique set of average values for at least the parameters n, β, t_o and ΔH. Armed with these values (plus the value of one additional parameter which we shall introduce in the next chapter) one could, in principle, predict the results of all experiments for each modality that involve only a single, constant stimulus, applied for a specified period of time. However, there are problems associated with the computation of these parameter values from published experimental data. Investigators conduct experiments using varied techniques, and do not always report all relevant experimental conditions. Moreover, if an investigator does not use a particular theory as a guide to experimental design, he or she will not always control all variables critical to the evaluation of the theory, or even report all relevant data. *Theory is as necessary to the experimenter as experimental results are to the theorist.*

It is exceedingly difficult to bring together experiments performed by different investigators at different epochs, under different sets of standards, and emerge from this amalgam with a consistent set of parameters for a given modality. However, we shall try. Let us consider the sense of taste of sodium chloride or saline solution. We have already analyzed several types of experiment involving the taste of sodium chloride solution:

(1) The "law of sensation" studies of Stevens (Figure 10.1) gave the value $n = 1.483$, using Equation (10.3). This value is a little higher than usually cited for n. We shall later use $n = 1.0$.

(2) The total number of jnd's for the sense of taste of sodium chloride for the 75% correct criterion can be estimated from the data of Holway and Hurvich using the usual Equations (3.11) or (3.12). We obtained the value of 9.35 jnd's, which seems rather small. Since the information per jnd is assumed to be constant, this information, ΔH, may be estimated from the equation

$$\Delta H \simeq \frac{\text{maximum information content of a stimulus}}{\text{total number of jnd's}} \simeq \frac{1.18}{9.35} = 0.126 \text{ n.u.}$$

That is, the value of the "channel capacity" for the sense of taste tends to be about 1.7 bits or 1.18 natural units of information.

We shall add several more parameter values in the next chapter.

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PREDICTION: EFFECT OF ADAPTATION ON EQUAL LOUDNESS CONTOURS

Adaptation is expected, from informational considerations, to produce divergence of equal loudness contours for reasons which can be seen directly from Equation (12.16). When loudness, F, is constant, then, from (12.16), since $H = F/k$ and $\Delta H = \Delta F/k$,

$$\Delta I \propto I\,\Delta F\,. \tag{12.36}$$

That is, if two tones are separated by a small constant loudness difference, ΔF, the corresponding difference in the physical intensity of the two tones will vary directly with the intensity of the lower tone (or, approximately with the mean physical intensity of the two tones). We can compare two sets of two tones, the first given to an unadapted ear and the second to an adapted ear, where both (mean) loudness, F, and difference in loudness, ΔF, is the same for both adapted and unadapted cases. Let I_u be the (mean) sound intensity for the unadapted ear, and I_a be the (mean) sound intensity for the adapted ear. Then, from Equation (12.36),

$$\Delta I_a \propto I_a \Delta F$$

and

$$\Delta I_u \propto I_u \Delta F\,.$$

Then, dividing these equations,

$$\Delta I_a / \Delta I_u = I_a / I_u\,. \tag{12.37}$$

But, by the definition of adaptation, $I_a > I_u$. That is, it requires greater sound intensity to produce loudness, F, in the adapted ear than in the unadapted ear. Therefore, $\Delta I_a > \Delta I_u$. That is, we can predict that equal loudness contours will diverge, with adaptation to sound[5].

In fact, measurements made by Békésy (1929) confirm the theory (Figure 12.3). Two tones of 800 Hz were presented to an unadapted ear. From his graph, we can determine that these tones were of sound pressures of about $8.0 \times 10^{-3}$and 7.0×10^{-2}dynes/cm^2. The ear was then adapted to a tone of high intensity and long duration at 800 Hz. Two more tones were then presented to the adapted ear, of sound pressures $3.2 \times 10^{-1}$and 9.8×10^{-1}dynes/cm^2. The loudness of these two tones matched the loudness of the previous two tones, so that both F and ΔF were equal. We notice, however, that

$$\Delta I_a = 9.8 \times 10^{-1} - 3.2 \times 10^{-1} = 6.6 \times 10^{-1}\,,$$

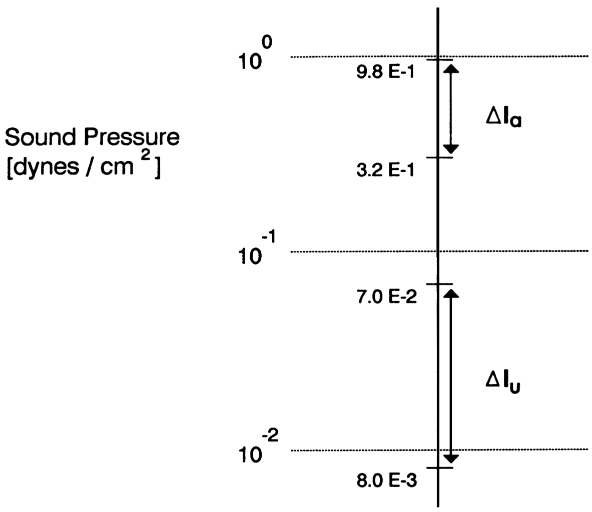

Figure 12.3 Data of Békésy (1929, Figure 1, at 800 Hz). ΔI_u and ΔI_a represent changes in sound pressure of a tone delivered to an unadapted and an adapted ear respectively. The lower boundary of ΔI_u sounds equally as loud as the lower boundary of ΔI_a, and the upper boundary of ΔI_u sounds as loud as the upper boundary of ΔI_a. That is, a tone at 8.0×10^{-3} dynes / cm^2, presented to the unadapted ear, sounds as loud as a tone of 3.2×10^{-1} dynes / cm^2, presented to the adapted ear. Similarly 7.0×10^{-2} dynes / cm^2 (unadapted) is as loud as 9.8×10^{-1} dynes/cm^2 (adapted). In units of dynes / cm^2, $\Delta I_a > \Delta I_u$.

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while

$$\Delta I_u = 7.0 \times 10^{-2} - 8.0 \times 10^{-3} = 6.2 \times 10^{-2} ,$$

where I is measured here in units of sound pressure. That is, $\Delta I_a > \Delta I_u$ as predicted.

SUMMARY

Amid the swirl of equations in this chapter, one tends to forget that our aim in this endeavor is not *this* algebraic relation or *that* numerical calculation, but rather to extract from the whole a tiny kernel of wisdom. The kernel probably centers around the concept of constant ΔH as an informational threshold, or informational differential limen.

The equation $F = kH$ did not play a large role in this chapter, making only a cameo appearance in Equations (12.7) – (12.9), and showing up for the finale on equal loudness contours. We used, nearly exclusively, Equation (12.1),

$$H = H(I, t) .$$

We introduced the condition that ΔH, a small, constant quantity of information, was necessary to make a distinction between two stimuli of different intensity. Using this theme in several variations, we were able to derive an expression for the Weber fraction, $\Delta I / I$, as a function of I and t. Equation (12.6) is probably its most practical form, giving the Weber fraction as a function of stimulus intensity. But Equation (12.16) has some theoretical interest in that the Weber fraction is shown to be a simple exponential function of stimulus entropy.

Through the medium of the theory of the Weber fraction, we were able to derive the Poulton-Teghtsoonian law, nearly from first principles, including the value of the constant. We were also able to demonstrate a property of equal loudness contours. We learned that while the jnd does not have a unique magnitude, if multiplied by a balancing factor, it does give rise to an invariant form.

The informational differential threshold, ΔH, can, by its definition, be "stacked." That is $N \times \Delta H =$ channel capacity, where N is equal to the total number of jnd's.[4]

We observed in this chapter, for a second time, that the power function exponent, n, has appeared in equations (for the Weber fraction) that have, ostensibly, nothing to do with the power law of sensation. (Where was the first occasion when n appeared outside of the power law?[6])

Since this chapter was largely bereft of the variable, F, there was no need to appeal to experiments in which sensation was measured numerically. Although the exponent, n, was in that manner born, it has now been shown to be a parameter which can be measured without reference to magnitude estimation. As we proceed, we shall find n arising again and again in expressions having nothing to do with subjective magnitude: equations for simple reaction time, for threshold detection, etc.

We derived, from the primary H-function, Equation (9.19) / (12.1), expressions for the Weber fraction, bringing now three classes of sensory law under the umbrella of this fundamental equation; the law of sensation, the principle of adaptation, and now the differential threshold. We completed the analogs with the ideal gas law. However, we shall continue, now, to derive still more sensory laws from the seminal Equation (9.19) / (12.1).

NOTES

1. The equation is derived neurophysiologically as Eq (1.1) of Lehmann's book (in German), and appears in another context on page 250.

2. By calculating $R =$ jnd's beneath plateau / total jnd's, the reader can establish for himself/herself that $R \geq 0.7$. In fact R is often greater than 0.85. There is, admittedly, a subjective element involved in the calculation. It depends on where an ill-defined plateau is considered to begin and end, as well as on just how great the physiologically maximum stimulus is taken to be.

3. The reader may also have noticed that the $F = kH$ relation has not been used hitherto in this chapter, except briefly in Equations (12.7) – (12.9). ΔF is, of course, equal to $\Delta H / k$, so we have replaced Fechner's $\Delta F =$ constant by $\Delta H =$ constant.

4. We have used the relation $\Delta H = H_{\max} / N_{\max}$, which is an approximation. That is, ΔH multiplied by the number of such ΔH's ($N_{\max}$ of them) is approximately equal to the total range of H, zero to $H_{\max}$.

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5. In Figure 1 of their chapter, Keidel *et al*. (1961) indicate the opposite: "Steps of equal loudness correspond to smaller steps of sound pressure in the adapted than in the undadapted ear." However, it is possible that the authors refer to steps measured in logarithmic coordinates.

6. The first occasion in which we encountered the power function exponent in a place other than the power law of sensation was when it appeared, unexpectedly, as a part of the Weber-Fechner law, Equation (10.5).

†. (2003 ed. note) In this section, I had confused modes in the first edition. So I have corrected the error, as best I can, in the second edition. Our calculation refers to the first of the two modes.

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